Detecting super-Nyquist-frequency gravitational waves using a pulsar timing array

Shu-Xu Yi (易疏序) & Shuang-Nan Zhang (张双南)

高能物理研究所

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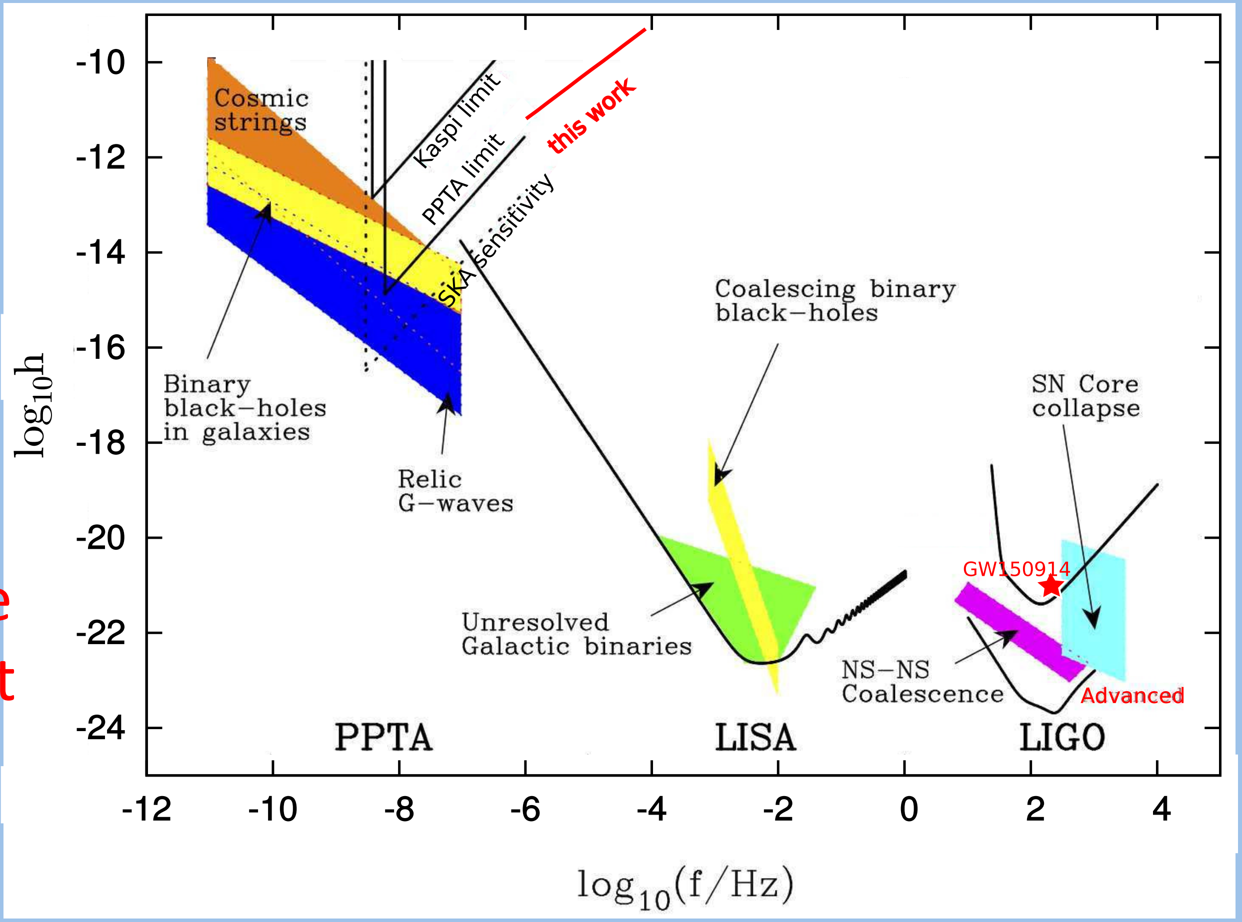
Upper frequency limit of Pulsar timing

The upper frequency limit of pulsar timing is set by the cadence of observation; ~10^-7 Hz

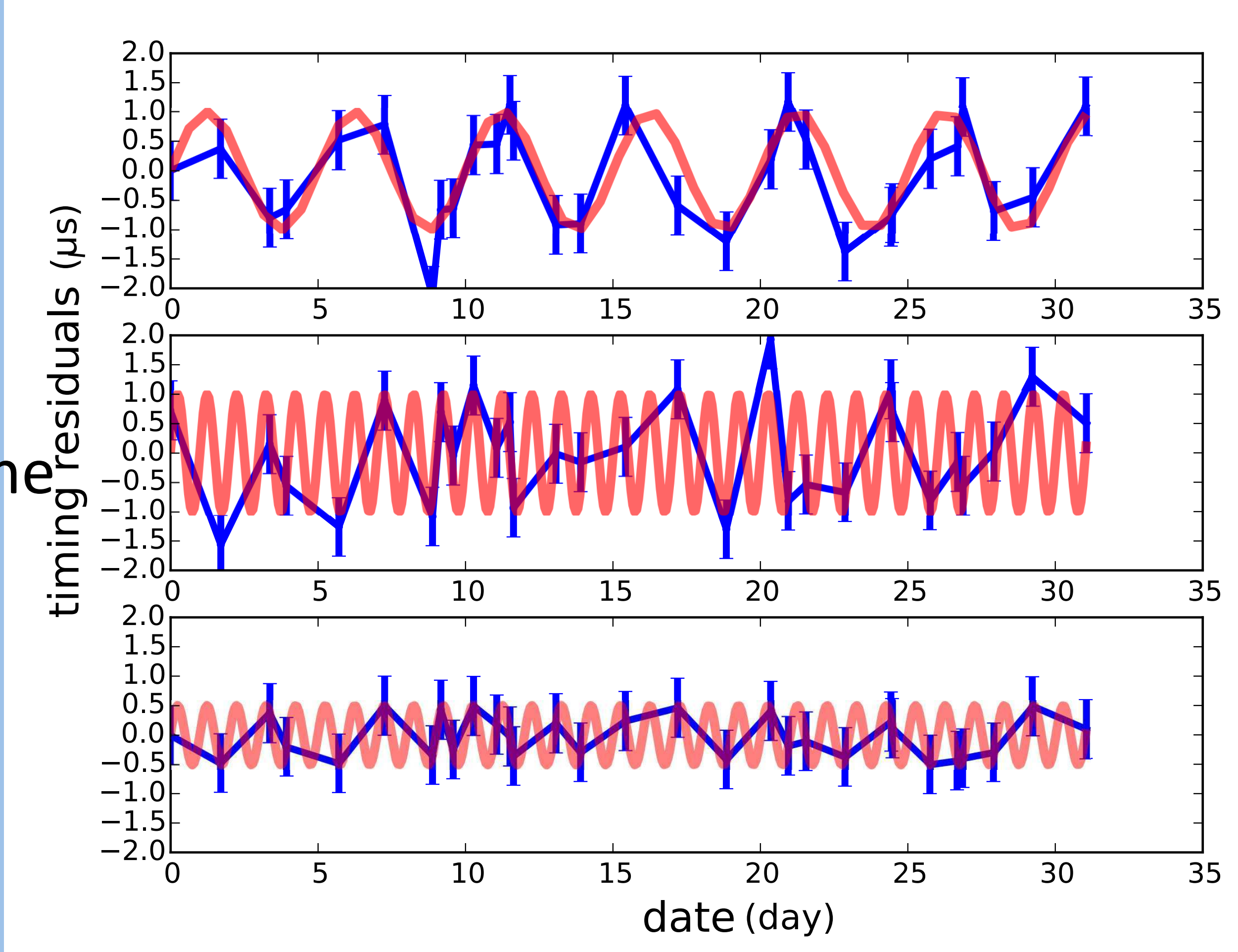
Detect GW above the upper frequency limit without increase cadence?



Thrane, E., & Romano, J. D. 2013

Upper frequency limit of Pulsar timing

under-sampling the signal will lost the information of the phases, but remain the information of the amplitude.



GW induced timing noise

timing residuals:

$$r(t) = F_+ A_+(t) + F_\times A_\times(t),$$

Geometric factors:

$$F_+ = \frac{1}{4(1-\cos\theta)}[(1+\sin^2\beta_s)\cos^2\beta\cos 2(\lambda_s-\lambda) - \sin 2\beta_s \sin 2\beta \cos(\lambda_s-\lambda) + \cos^2\beta_s(2-3\cos^2\beta)],$$

$$F_\times = \frac{1}{2(1-\cos\theta)}[\cos\beta_s \sin 2\beta \sin(\lambda_s-\lambda) - \sin\beta_s \cos^2\beta \sin 2(\lambda_s-\lambda)],$$

plus mode and cross mode:

$$A_+ = h/\omega[(1+\cos^2\iota)\cos 2\phi \sin\omega t + 2\cos\iota \sin 2\phi \cos\omega t],$$

$$A_\times = h/\omega[(1+\cos^2\iota)\sin 2\phi \sin\omega t - 2\cos\iota \cos 2\phi \cos\omega t].$$

GW induced timing noise

$$r(t) = \mu K \xi \sin(\omega t + \psi).$$

$$\mu^2 = F_+^2 + F_\times^2$$

$$K = h/\omega$$

the relative position between pulsar and the GW source

$$\xi^2 = ((1 + \cos^2 \iota) \sin \gamma)^2 + (2 \cos \iota \cos \gamma)^2$$

the property of GW

$$\sigma_{\mathrm{GW}}^2 = \frac{1}{2}\mu^2 K^2 \xi^2.$$

White noise variance as a function of pulsar coordinates

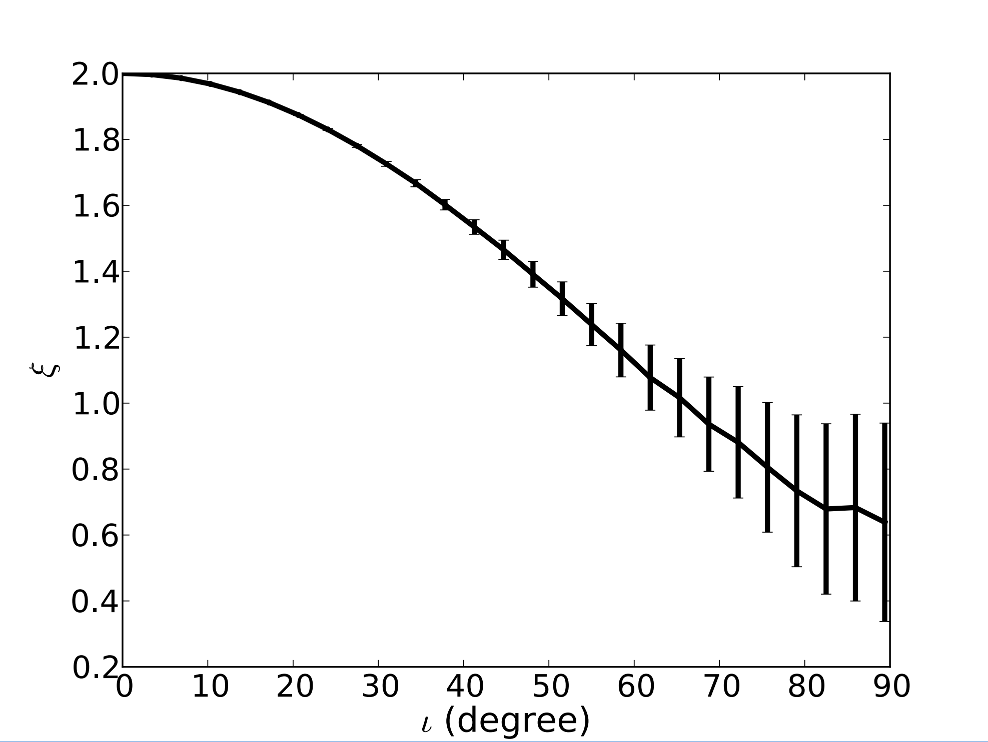
$$r(t) = \mu K \xi \sin(\omega t + \psi).$$

$$\xi^2 = ((1 + \cos^2 \iota) \sin \gamma)^2 + (2 \cos \iota \cos \gamma)^2$$

$$\gamma = \arctan(F_+/F_\times)$$

intrinsic scatter about the
pure proportional relationship

$$\sigma_{\mathrm{GW}}^2 = \frac{1}{2}\mu^2 K^2 \xi^2.$$



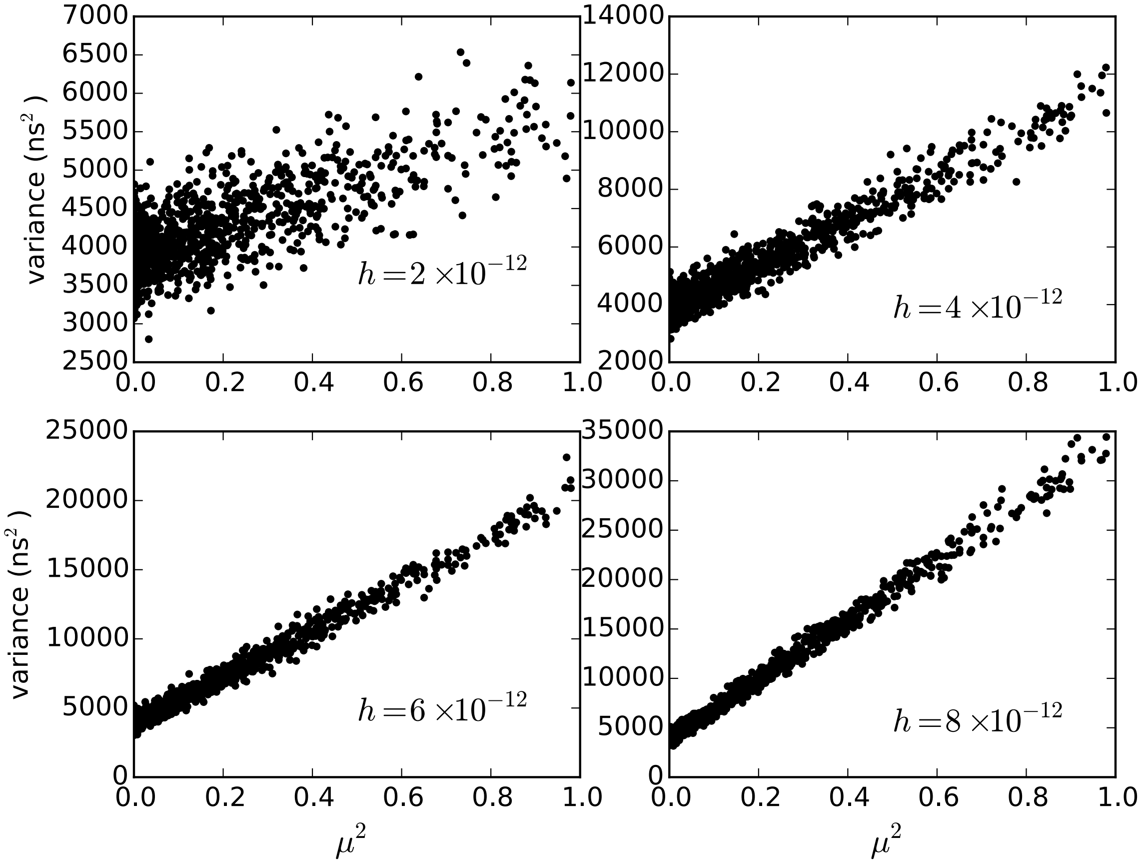
test with simulated data

$$r(t_i) = G(0, \alpha) + G(0, E(t_i)) + \mu K \xi \sin(2\pi f t_i),$$

intrinsic white noise: 50 ns;

cadence and TOA uncertainty: according to PSR J0437-4715 in PPTA dr1.

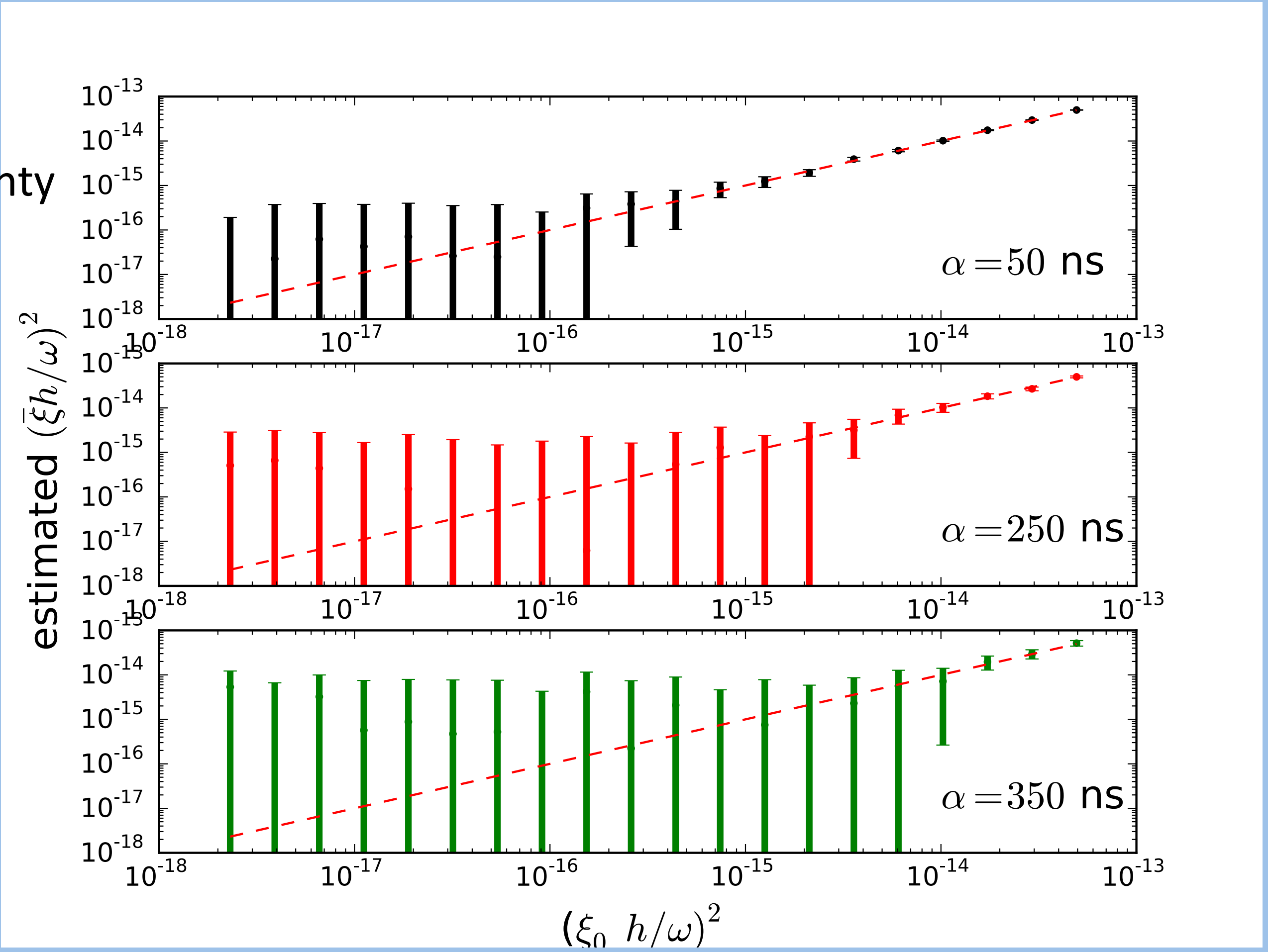
$$\sigma^2_{\mathrm{GW}} = \frac{1}{2}\mu^2 K^2 \xi^2.$$



test with simulated data

Below a critical h, the uncertainty of the estimated too large.

This critical h increases as the intrinsic white noise increases.



test with real data

- Total noise=

  red noise + white noise

  - noise on account of TOA error bar +
  - noise not on account of TOA errorbar: σ^2_{remain}

    =SNGW induced + intrinsic

    - EFAC and EQUAD in tempo2

test with real data-selecting the pulsars

selected from
PPTA DR1 &
NANOgrav dfg+12

name	$\sigma_{\rm remain}$ (ns)	name	$\sigma_{\rm remain}$ (ns)
J0437-4715	42	J1857+0943	8
J0613-0200	19	J1909-3744	8
J0711-6830	3	J2124-3358	8
J1024-0719	7	J2129-5721	5
J1045-4509	15	J2145-0750	3
J1600-3053	9	J1012+5307	11
J1603-7202	24	J1640+2224	10
J1643-1224	3	J1910+1256	17
J1713+0747	9	J2317+1439	31
J1730-2304	4	J0030+0451	3
J1732-5049	2	J1853+1308	2
J1744-1134	6	J1918-0642	9
J1824-2452A	13		

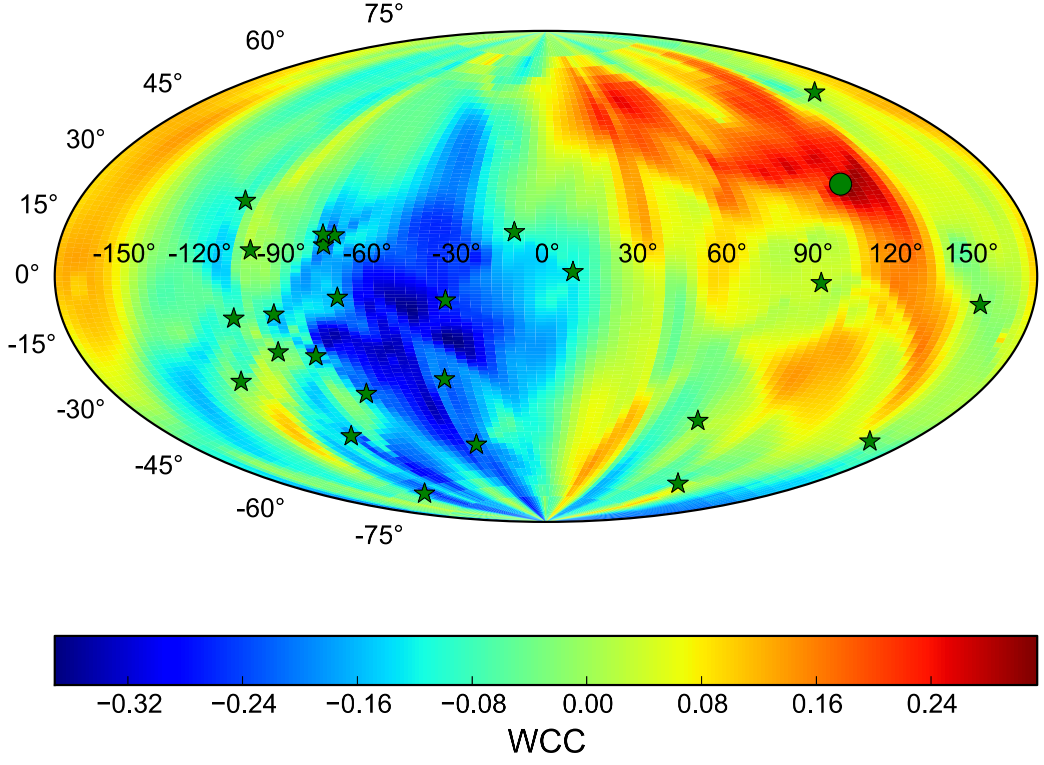
Detecting the SNGW

1, we divide the celestial sphere into 100*100 equal-area grids. For each grid we suppose that the GW source is locates within it and we calculate μ^2 for all pulsars.

2, Calculate the Pearson correlation coefficient between σ^2_{remain} and μ^2

Detecting the SNGW

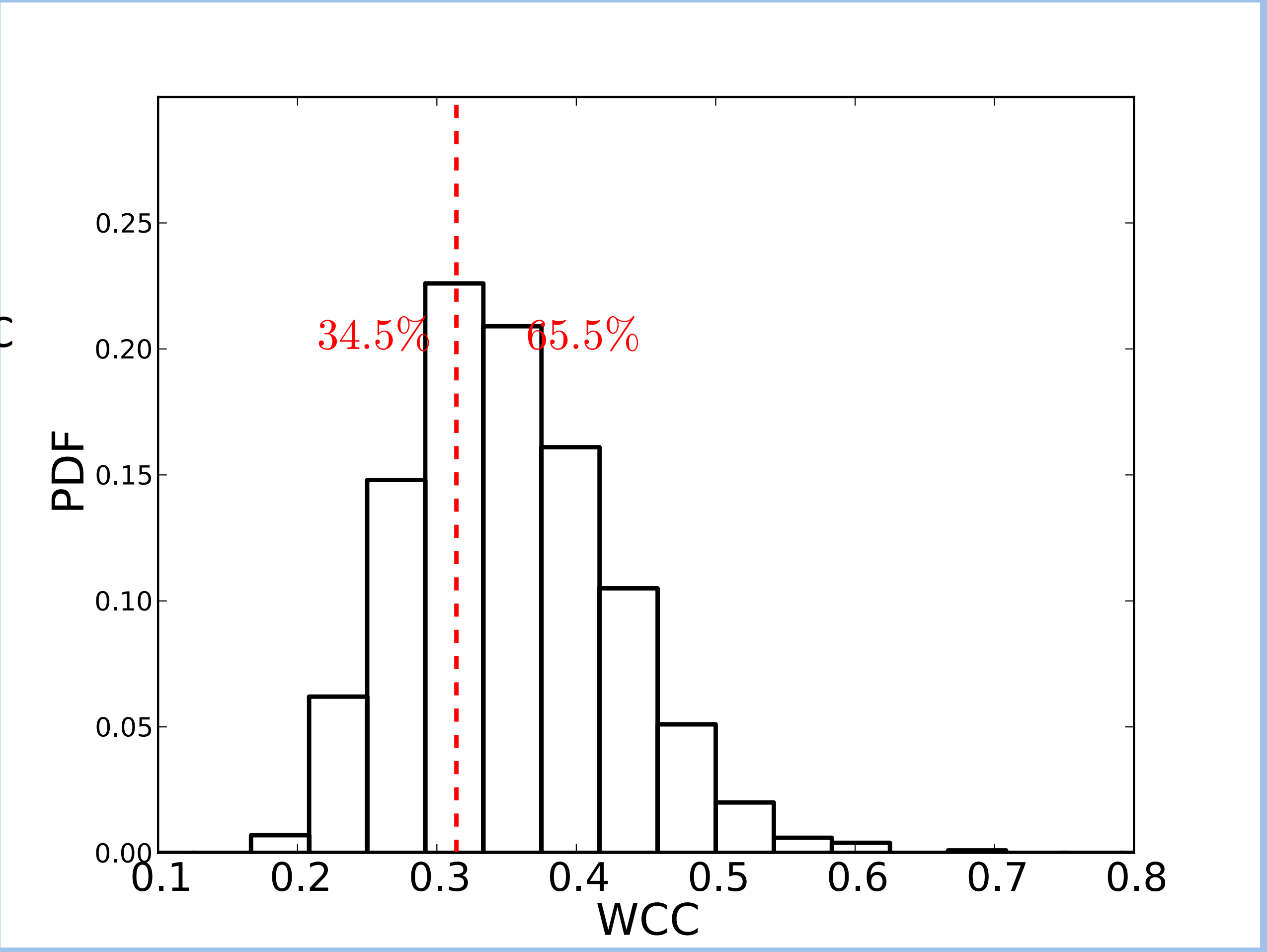
Sky map of Pearson Correlation coefficient between between σ^2_{remain} and μ^2



Green dot denotes the position where the correction is max randomly shuffle σ^2_{remain} of pulsars for 1,000 times.

Calculate the skymap of PCC

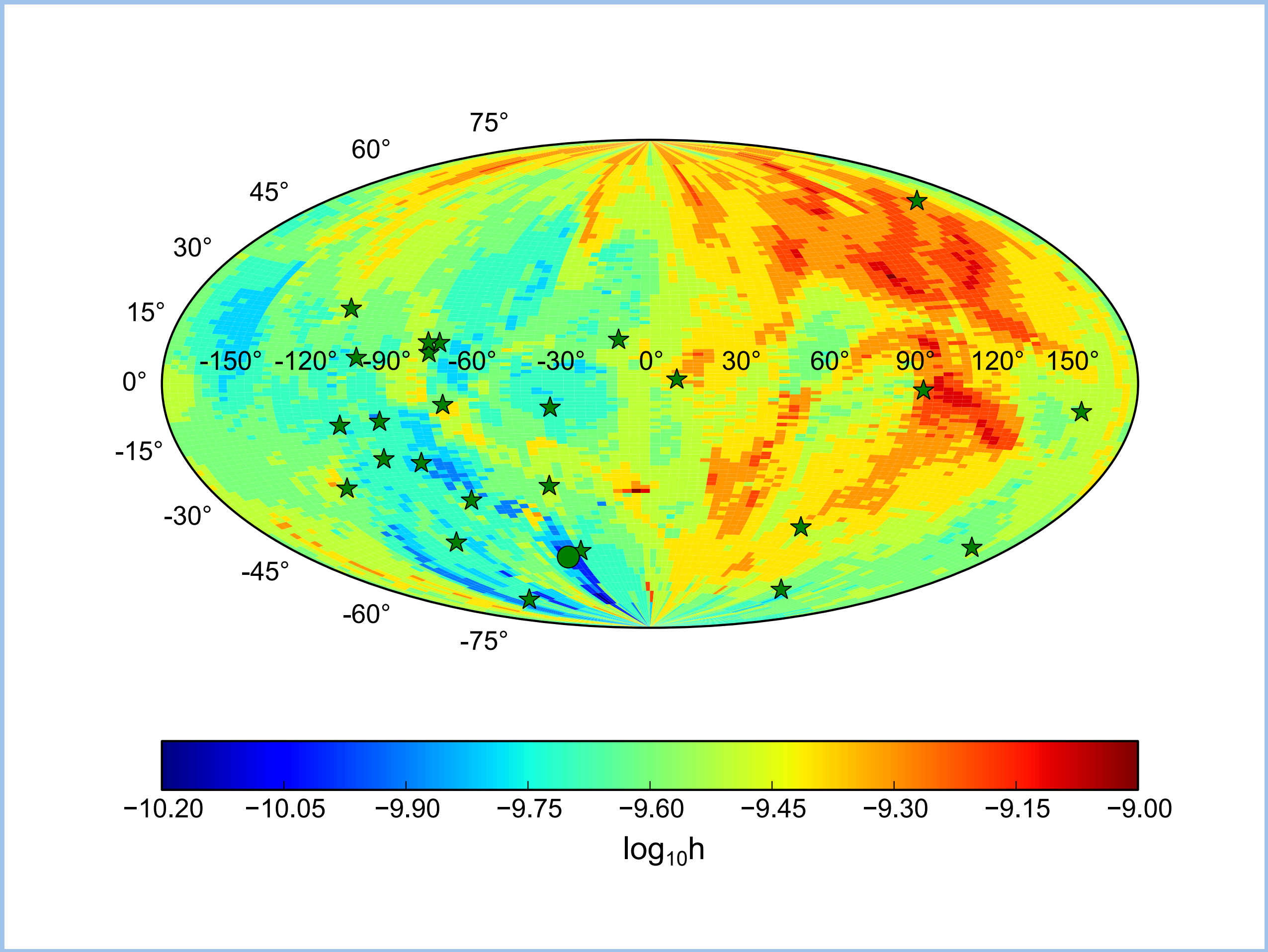
Find the largest number for each permutation.



Sensitivity to single SNFGW sources

- Divide the sky sphere uniformly into 100*100 grids. Put SNFGW source in each grid.
- Starting from a small GW strain h value and a random polarization angle , generate a series of timing residuals
- Follow the SNFGW source-detecting procedure described in the above section. Increase h and return to step 2, until the detection significance reaches 99%.
- Record the current value of h as the minimum GW strain that the dataset is sensitive to. Move to the next grid point of the sky sphere.

Sensitivity to single SNFGW sources



Conclusions

- SNFGWs leave additional white noise in the timing residuals
- $\sigma^2{}_{GW}$ proportional to μ^2, scaled by $1/2(\xi h/\omega)^2$
- f_{GW} given, stronger GW give more significant $\sigma 2_{GW}$-μ^2 relationship
- Intrinsic white noise affects this method
- Skymap of sensitivity